Discrete Mathematics

Books :-

- Kenneth H. Rosen
- Descrete Mathematics and its applications with combinatorics and graph theory, Tata Maccraw Hill
- C.L. Liu
- Elements of discrete mathematics
- Edgar G Goodaire, Michael M. Parameter, Discrete mathematics with graph theory.

Units :-

1. Logic and proposition
2. Proof methods
3. PERT & CPM
4. Graph theory
5. Cryptography, Permutation & Combination

Discrete mathematics is the study of mathematical structure that are fundamentally discrete rather then contienous as in the case of analysis and calculas. It includes integers, graphs, logics which have seprated distinct values.

Logic is the study of method of reasoning.

Propositions is a sentence which is either true or false and the true and false value is known as truth values of proposition

eg:- eg of not proposition :- ① what time it is ?
② Close the door
③ $x > 5$

eg of proposition :- ① apple is red - T
② Mango is blue - F
③ $8 > 5$

→ Compound Proposition :- Combination of 2 or more proposition

eg :- $8 > 5$ and $6 + 3 = 5$

→ Logical operators are used to form new propositions from two or more existing propositions.

→ Truth table :- It displays the truth value of compound statement in term of its component parts.

① Negation (NOT) ($\sim$) :- If p is a statement then negation of p is the statement 'it is not the case that p' and it is written as $\sim p$.

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Truth table:

p	$\sim p$
T	F
F	T

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Q i) Let p and q be the two propositions
q: you do every exercise of this book
p: you can get A in final exam
write the following propositions p and q and logical connections.

p: you get A in final exam and you do every exercise of this book.

⇒ p: A
q: you do

$p \wedge q$

ii) If you do not get A in the final exam then you will do every exercise of this book

⇒ $\sim p \rightarrow q$

iii) You do not get A in the final exams

⇒ $\sim p$

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② Conjunction (AND)
Let p and q be two proposition. The conjunction of p and q is the proposition 'p and q' and it is written mathematically as $(p \wedge q)$

p and q is true when both are true
The truth table of p and q is defined as

p	q	$p \wedge q$
T	T	T
T	F	F
F	~~T~~	F
F	F	F

Eg. p: It is cloudy
q: It is raining now

$p \wedge q$: It is cloudy and it is raining now

③ Disjunction (OR) $\vee$
Let p and q be the two propositions, then disjunction of p and q is written as 'p OR q' mathematically as $p \vee q$

p OR q is true, when atleast one is true.
Inclusive.

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p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Inclusive

p: Jaipur is in rajasthan
q: Jaipur is in India

$p \vee q$: Jaipur is in Rajasthan or Jaipur is in India.

Exclusive

Exclusive OR. $\oplus$

p ~~exclusive OR~~ q is

$p \oplus q$ is true when exact one is true.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Eg.

p: 3 is an odd number
q: 3 is an even number

$p \oplus q$: 3 is an odd number OR 3 is an even number.

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④ Implication (→)
Let p and q be the two propositions then
Implication of p and q is defined as
'If p then q'.
mathematically : $p \to q$

$p \to q$ is false when p is true and
q is false

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

p : f(x) is an differentiable function
q : f(x) is a continous function

True $p \to q$: If f(x) is an differentiable function
then it is continuous.

⑤ Biconditional (↔)
Let p and q be the two proposition then
by biconditional of p and q is defined as
'p iff q', 'p if and only if q'
mathematical : $p \leftrightarrow q$

p if and only if q is trues when both
p & q have same truth value.

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p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

eg: Let A be a matrix
p : Inverse of A exists
q : $|A| \neq 0$

True $p \leftrightarrow q$: Inverse of A exists if and only if $|A| \neq 0$.

Q. Which of the following are propositions

i) Answer these questions
→ ~~Ex~~ ~~A~~ X bcoz its truth value doesn't exist

ii. $7 + 3 = 11$
→ ✓

iii If stock prices fall then shyam will loss money
✓

iv There is an Integer x such that $x^2 = 3$
→ ✓

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Q Give the negition of all statements

i) $5+8>3$
$5+8<3$

ii. 2 is an even integer or 8 is an odd integer
→ 2 is not an even integer or 8 is not an odd integer.

iii.
Q Determine whether these conditional and biconditional are true or false

i) If $1+1=2$ then $2+2=5$
→ By table of implication value of this proposition is false

ii. $0>1$ iff $2>1$
→ Bcoz both value are different, so value is false

Q Let p and q be the two proposition
p: Today is monday
q: Grass is dry
Express each of the proposition as an english sentence.
$\sim p \rightarrow \sim q$
→ If today is not monday then grass will not dry.

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$\sim p \wedge \sim q$

⇒ Today is not monday and grass is not dry

$\sim p \vee (p \wedge q)$

⇒ Today is not monday or Today is monday and grass is dry.

Q Construct the truth table

$(p \wedge q) \rightarrow p \vee q$

p	q	$p \wedge q$	$p \vee q$	$p \wedge q \rightarrow p \vee q$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

$(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$

p	q	$\sim q$	$\sim p$	$p \rightarrow q$	$\sim q \rightarrow \sim p$	$(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$
T	T	F	F	T	T	T
T	F	T	F	F	F	T
F	T	F	T	T	T	T
F	F	T	T	T	T	T

Converse, Inverse, Contrapositive

Let p and q are the two proposition and $p \rightarrow q$ is a compound proposition

Converse → $q \rightarrow p$

Inverse → $\sim p \rightarrow \sim q$

Contrapositive → $\sim q \rightarrow \sim p$

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Contrapositive.

Converse

Inverse

p	q	~q	~p	~q → ~p	q → p	~p → ~q
T	T	F	F	T	T	T
T	F	T	F	F	T	T
F	T	F	T	T	F	F
F	F	T	T	T	T	T

Q State the converse, inverse & contrapositive of this implication
- If you drive more than 10 km then you will need to buy petrol

p: You drive more than 10 km
q: You will need to buy petrol

Converse If You will need to buy petrol then you will drive more than 10 km

Inverse. If You do not drive more than 10 km then you ~~don~~ will not need to buy petrol

Contrapo. - If you ~~will~~ do not need to buy petrol then you ~~do~~ will not drive more than 10 km.

Tautology.

A compound proposition that is always true for any assignment of truth values to the propositional variable is called tautology.

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Contradiction / Fallacy

A compound proposition that is always false for any assignment of truth values to the propositional variable is called fallacy.

Contingency

A compound proposition i.e. neither tautology nor fallacy is called as contingency.

Q Show that the following proposition is a tautology.

$$(p \to q) \wedge (q \to r) \longrightarrow p \to r$$

p	q	r	$p \to q$	$q \to r$	$p \to r$	$(p \to q) \wedge (q \to r)$	$(p \to q) \wedge (q \to r) \to p \to r$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	T
F	T	T	T	T	T	T	T
T	F	F	F	T	F	F	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

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Q Is $p \wedge (q \wedge \sim p)$ fallacy?

→

p	q	$\sim p$	$(q \wedge \sim p)$	$p \wedge (q \wedge \sim p)$
T	T	F	F	F
T	F	F	F	F
F	T	T	T	F
F	F	T	F	F

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Propositional function

Eg $p(x) : x > 3$

It is a function of x which can be made proposition

There are two ways to change a propositional function into a proposition

(1) Assign it a value from the universe of discourse

For this eg. let Z be the universe of discourse.

(2) Provide a quantification.

Quantifier: A quantifier is an operator that limits the variable of the proposition

i) Universal quantifier ($\forall$)

$\forall x \; P(x) : x > 3$ is true when it is true for all values of x of universe of discourse

ii) Existential Quantifier ($\exists x$)

$\exists x \; p(x) : x > 3$

There exist $x \; P(x)$, is true, when it is true for atleast one value of x of universe of discourse

Eg. $5 \in Z$

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Q. What is the truth value of $\forall x (x^2 \geq x)$ if D (universe of discourse) if D consist of all real no. and what is the truth value if D consist of an integer.

→ (1) $x^2 \geq x$

$x^2 - x \geq 0$

$x(x-1) \geq 0$

$x \geq 0, \quad x-1 \geq 0$

$x \geq 0, \quad x \geq 1 \Rightarrow x \geq 1$

(2) $x \leq 0, \quad x-1 \leq 0$

$x \leq 0, \quad x \leq 1 \Rightarrow x \leq 0$

It is true when

$x \geq 1, \quad x \leq 0$

i) when D is real no. → False

bcoz it is not true for no. b/w 0 & 1

so truth value is false.

ii) D is Integer

Bcoz there is no integer b/w 0 & 1

∴ its truth value is true.

Q. Translate each one of this statement into logical expression using Quantifier and logical connectives.

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- Not everyone is perfect
- All your friends are perfect.
- One of your friend is perfect
- Everyone is your friend and perfect

$p(x)$: x is perfect
$q(x)$: x is your friend.

$\sim \forall_x\, p(x)$

$\forall x\, q(x) \rightarrow p(x)$

$\exists x\; p(x) \wedge q(x)$

$\forall x\, q(x) \wedge p(x)$

$p \vee q$

Principle disjunctive normal form (PDNF)

$p \wedge q$

It is the disjunction of min terms.
OR

min term → It is the conjunction of propositional variables where each variable appear only once either in form of variable and its negation.

Let p & q be the two propositional variable then there will be $2^2 = 4$ min terms
if there are n variables then no. of min terms
2^n

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Principle Conjunctive normal form (PCNF)

It is the conjunction of max terms

max terms - It is the disjunction of propositional variables where each variable appears only once either in form of variable or its negation

$p \vee q \vee r$

If there are n variables then max terms will be 2^n

Q Find the PDNF and PCNF of $(\sim p \to r) \wedge (p \leftrightarrow q)$

→

p	q	r	$\sim p$	$\sim p \to r$	$p \leftrightarrow q$	$(\sim p \to r) \wedge (p \leftrightarrow q)$
T	T	T	F	T	T	T
T	T	F	F	T	T	T
T	F	T	F	T	F	~~T~~ F
F	T	T	T	T	F	~~T~~ F
T	F	F	F	T	F	~~T~~ F
F	T	F	T	F	F	~~T~~ F
F	F	T	T	T	T	T
F	F	F	T	F	T	~~T~~ F

PDNF : disjunction of minterms.

minterms (I row) = $p \wedge q \wedge r$

II row = $p \wedge q \wedge \sim r$

III = $\sim p \wedge \sim q \wedge r$

$= (p \wedge q \wedge r) \vee (p \wedge q \wedge \sim r) \vee (\sim p \wedge \sim q \wedge r)$

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PCNF = Conjunction of max. term.

III row $= \sim p \vee q \vee \sim r$

IV $= p \vee \sim q \vee \sim r$

V $= \sim p \vee q \vee r$

VI $= p \vee \sim q \vee r$

VIII $= p \vee q \vee r$

$(\sim p \vee q \vee \sim r) \wedge (p \vee \sim q \vee \sim r) \wedge (\sim p \vee q \vee r)$
$\wedge (p \vee \sim q \vee r) \wedge (p \vee q \vee r)$

Proof methods.

All mathematical theorem are composed of implication of the type $p_1 \wedge p_2 \dots \wedge p_n \to q$

~~Write the~~ special case : $p \to q$.

Direct method.

Write the rule of inference for direct method
To construct a proof of the statement for all x, $x \in D$, D is universe of discourse
We start by selecting an arbitary element a of the domain (universe of discourse) and show that $p(a) \to q(a)$ is true

By assuming that $p(a)$ is true and we show that $q(a)$ will be true

Q Prove the theorem if x is an odd integer than x^2 is odd integer.

→ $p(x)$: x is an odd integer
$q(x)$: x^2 is an odd integer

$\forall x \; p(x) \to q(x)$
$p(a) \to q(a)$
$a = 2n+1 \qquad n \in \mathbb{Z}$
$a^2 = (2n+1)^2$
$= 4n^2 + 1 + 4n$
$= 2(2n^2 + 2n) + 1$

$\therefore a^2$ is odd integer

$p(a) \to q(a)$ is true.

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Q Give a direct prove to show that for all integer n if $(n-2)$ is divisible by 3. Then n^2-1 is divisible by 3.

→ ~~proof~~ $p(n)$: $n-2$ is divisible by 3
$q(n)$: n^2-1 is divisible by 3.

$\forall n \; p(n) \to q(n)$

$p(a) \to q(a)$

Let us assume $p(a)$ is true

$n-2 = 3k \rightarrow (n = 3k+2)$ $k \in \mathbb{Z}$

$= n^2-1$
$= (3k+2)^2 - 1$
$= 9k^2 + 4 + 12k - 1$
$= 3(3k^2 + 4k + 1)$

$p(a) \to q(a)$ is true

∴ Direct proof method it is true.

Q Proof the for all condition x & y if x & y are odd then the product $x \cdot y$ is odd.

$p(x) \to x$ is an odd integer.
$q(y) \to y$ is an odd integer.

$r(x,y)$ is odd integer

$\forall xy \; p(x) \wedge q(y) \to r(x,y)$

$p(a) \wedge q(a) \to r(a)$

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$x = 2m+1$, $y = 2n+1$

$xy = (2m+1)(2n+1)$

$= 4mn + 2m + 2n + 1$

$= 2(2mn + m + n) + 1$

It is a multiple of 2 and odd.

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Indirect Proof Method

i) Contrapositive

Since the implication $p \to q$ is equivalent to $\sim q \to \sim p$ i.e. contrapositive of $p \to q$. Thus to proof the implication $p \to q$ we will proof $\sim q \to \sim p$ and to proof it we assume not q is ~~to~~ true and show that $\sim p$ ~~is~~ will be true.

Give an indirect proof of the theorem if $3n+2$ is odd then n is odd.

p : $3n+2$ is odd

q : n is odd

$\forall n \; p \to q$ n is integer.

we will prove $\sim q \to \sim p$

Let us assume

n is even

$n = 2k$ $k \in \mathbb{Z}$

to show $3n+2$ should be even.

$(3n+2)$

$3(2k)+2$

$6k+2$

$2(3k+1)$

It is a multiple of 2 so it is a even no.

$\sim p$ is true

$\therefore \sim q \to \sim p$ is true.

ii. Contradiction :-

In this method we assume $\sim p$ is true and arrive at the contradiction.

Q. Show $\sqrt{2}$ is irrational.

p : $\sqrt{2}$ is irrational

Let us assume $\sim p$ is true

$\sim p$: $\sqrt{2}$ is rational no.

$$\sqrt{2} = \frac{a}{b}$$

where a/b have no common factor other than 1, $a, b \in Z$

squaring both side

$$2 = \frac{a^2}{b^2}$$

(square of even no. is even)

$$a^2 = 2b^2 \quad \text{— (1)}$$

$\therefore a^2$ is an even no., a is even

$$a = 2K \quad \text{— (2)}$$

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putting a in (1)

$4k^2 = 2b^2$

$2k^2 = b^2$

∴ b^2 is even, b is even

now a & b are even

∴ They have common factor 2.

But for the rational there should be no common factor other than one

∴ It is a contradiction.

∴ Assumption is wrong

It means it is irrational no.

(3) Mathematical Induction

Let p be a theorem for all natural no.

i) Basis step:
Showing that $p(n_0)$ is true [$n(n+1)$]

ii) Inductive hypothesis → Assume $p(k)$ is true

iii) Inductive step → We will show $p(k+1)$ is true by showing that $p(k)$ is true

Q Show that $n < 2^n$ for all positive integer

→ i) Basis step: to prove $n = 1$.

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$1 < 2^1$

$1 < 2$

true.

ii. Inductive hypothesis →
Assume $p(k)$ is true $(k \geq 1)$

$\boxed{n < 2^n = p(n)}$

$k < 2^k$ — (1)

iii. Inductive step. To prove $p(k) \to p(k+1)$
to show $k+1 < 2^{k+1}$

by (1) $(k < 2^k)$

$k+1 < 2^k + 1$
$< 2^k + 2^k$
$< 2^k (1+1)$
$k+1 < 2^{k+1}$

Q Proof the theorem by mathematical Induction.

$\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \dots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$ $n \to N$

→ i) Basis step → for $n = 1$

$\frac{1}{1\cdot 3} = \frac{1}{3}$

by $\frac{n}{2n+1} = \frac{1}{3}$ $(n=1)$

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ii. Inductive hypothesis:

Assume $p(k)$ is true

$$\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \cdots + \frac{1}{(2k-1)(2k+1)} = \frac{k}{2k+1}$$

iii. Inductive step: $p(k) \to p(k+1)$

To show $p(k+1)$ is true

$$\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \cdots + \frac{1}{(2k-1)(2k+1)} + \frac{1}{[2(k+1)-1][2(k+1)+1]}$$

$$= \frac{k+1}{2(k+1)+1}$$

LHS.

$$\Rightarrow \frac{k}{2k+1} + \frac{1}{[2(k+1)-1][2(k+1)+1]}$$

$$\frac{k}{(2k+1)} + \frac{1}{(2k+1)(2k+3)}$$

$$\frac{k(2k+3)+1}{(2k+1)(2k+3)}$$

$$\frac{2k^2+3k+1}{\cancel{4k^2+6k+2k+3}\ (2k+1)(2k+3)}$$

$$\frac{2k^2+2k+k+1}{(2k+1)(2k+3)}$$

$$\frac{\cancel{(2k+1)}(k+1)}{\cancel{(2k+1)}(2k+3)} = \left(\frac{k+1}{2k+3}\right) = \text{RHS.}$$

HP

P $1 + 2 + 3 + n = \frac{n(n+1)}{2}$

Q) $1^2 + 2^2 + 3^2 + \ldots$

Q. $1 + 2 + 2^2 + \ldots + 2^n = 2^{n+1} - 1$

$n \geq 0$

i) ⇒ Basis step $n = 0$

$2^0 = 2^{0+1} - 1$

$1 = 2 - 1 = 1$

ii) Inductive hypothesis
Assume $p(k)$ is true

$1 + 2 + 2^2 + \ldots + 2^k = 2^{k+1} - 1$ — (1)

To show $p(k+1)$ is true ⇒ $1 + 2 + 2^2 + \ldots + 2^k + 2^{k+1} = 2^{k+2} - 1$

iii) Inductive step
$p(k) \rightarrow p(k+1)$

$=$

$2^{k+1} - 1 + 2^{k+1}$

$2^{k+1}(1+1) - 1$

$2^{k+2} - 1$

by principle of mathematical Induction, this theorem is true.

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Q. Show that for all positive integer $3^{2n+1}+2^{n+2}$ divisible by 7.

① Basis step.

for ~~n = 0~~ $n = 1$

$= 3^{2\times 0+1} + 2^{0+2}$ $= 3^{2\times 1+1} + 2^{1+2}$

$= 3 + 4 = 7$ $= 35$

P(n): It is divisible by 7.

② Inductive hypothesis.

Assume p(k) is true

$\Rightarrow 3^{2k+1} + 2^{k+2}$ is divisible by 7.

$3^{2k+1} + 2^{k+2} = 7m$ $m \in \mathbb{Z}$

③ → Inductive step.

$$p(k) \longrightarrow p(k+1)$$

To prove it we assume p(k) is true and will show p(k+1) true.

$$p(k+1) = 3^{2(k+1)+1} + 2^{(k+1)+2}$$
$$= 3^{2k+3} + 2^{k+3}$$
$$= 3^{2k+1}\, 3^2 + 2^{k+2} \cdot 2$$
$$= 3^{2k+1}\, 9 + 2^{k+2} \cdot 2$$
$$= 3^{2k+1}(7+2) + 2 \cdot 2^{k+2}$$
$$= 7 \cdot 3^{2k+1} + 2(3^{2k+1} + 2^{k+2})$$
$$= 7 \cdot 3^{2k+1} + 2 \cdot 7m \quad \text{(using 1)}$$
$$= 7(3^{2k+1} + 2m) = 7r$$

It is the multiple of 7.

∴ P(k+1) divisible by 7.

∴ p(k) → p(k+1) is true.

∴ by mathematical induction is valid for positive integer

Q1. Find the PCNF & PDNF of the following compound proposition. $\sim(p \vee q) \leftrightarrow (p \wedge q)$

⇒

p	q	$p \vee q$	$\sim p \vee q$	$p \wedge q$	$\sim p \vee q \leftrightarrow p \wedge q$
T	T	T	F	T	F
T	F	T	F	F	T
F	T	T	F	F	T
F	F	F	T	F	F

PDNF

minterm II → $p \wedge \sim q$

III → $\sim p \wedge q$

$(p \wedge \sim q) \vee (\sim p \wedge q)$

PCNF

max term I → $\sim p \vee \sim q$

IV → $p \vee q$

$(\sim p \vee \sim q) \wedge (p \vee q)$

Q② Show that the following compound proposition is logically equivalent. $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

$p \leftrightarrow q$	p	q	$p \wedge q$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$(p \wedge q) \vee (\sim p \wedge \sim q)$
T	T	T	T	F	F	F	T
F	T	F	F	F	T	F	F
F	F	T	F	T	F	F	F
T	F	F	F	T	T	T	T

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In above table last two columns are same which are corresponding to L.H.S. & R.H.S.

Therefore $p \leftrightarrow q \equiv (p \wedge q) \wedge (\sim p \wedge \sim q)$

logically equivalent.

Q② $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

$p \leftrightarrow q$	p	q	$p \to q$	$q \to p$	$(p \to q) \wedge (q \to p)$
T	T	T	T	T	T
F	T	F	F	T	F
F	F	T	T	F	F
T	F	F	T	T	T

Q. Let p : he is old

q : he is clever

Write the following in symbolic form.

It is not true that he is young or not clever.

$\sim(\sim p \vee \sim q)$

Q Express the following statement into logical expression using predicate quantifier and logical connective

Every body must take DMS course or be a computer sci student

→ $p(x)$: x takes DMS course
$q(x)$: x is a computer sci student.

$\forall x \; p(x) \vee q(x)$

Q Let p be any proposition then $p \vee \sim p \equiv$

p	$\sim p$	$p \vee \sim p$
T	F	T
F	T	T

Q. Prove the theorem : x is odd integer iff x^2 is odd.

→ $p(x) \leftrightarrow q(x)$

$p(x) \rightarrow q(x)$ ii) $q(x) \rightarrow p(x)$

contrapositive

$\sim p(x) \rightarrow \sim q(x)$

$x = 2n+1$

$x^2 = 4$

$x = 2n$

$x^2 = 4n^2$

$= 2(2n^2)$